Optimal Incremental-Redundancy Strategy for Type-II Hybrid ARQ

Elisabeth Uhlemann†‡, Lars K. Rasmussen†‡, Alex Grant§ and Per-Arne Wiberg‡

†Computing and Commun. Laboratory, Halmstad University, Box 823, SE-301 18 Halmstad, Sweden.
‡Chalmers University of Technology, SE-412 96 Göteborg, Sweden.
§Inst. for Telecommunications Research in Telecommunications Research, University of South Australia, Mawson Lakes, SA 5095, Australia.

E-mail: bettan@ide.hh.se, lars.rasmussen@unisa.edu.au, alex.grant@unisa.edu.au, pelle@ide.hh.se

Abstract — Given a type-II hybrid ARQ scheme, we propose a general solution to find the optimal partitioning of \( n - k \) parity bits over at most \( M \) transmissions. The solution is in terms of maximizing the average code rate for a given puncturing pattern.

I. INTRODUCTION

Incremental redundancy (IR) hybrid ARQ type-II (IR-HARQ) schemes are appealing since all received observables are used in the decoding process [1]. Such schemes are commonly based on rate compatible punctured (RCP) codes, e.g., the RCP turbo code family [1]. Puncturing strategies for these code families have been optimized for maximizing the average code rate of the ARQ scheme, given an IR strategy. Here, an IR strategy is defined by a maximum number of allowed transmissions and the number of check bits to be included in each transmission. In contrast, we suggest an approach for finding the optimal IR strategy for maximizing the average code rate, given a puncturing pattern.

II. OPTIMAL IR STRATEGY

A data frame is a set of \( k \) information bits passed through a rate \( k/n \) encoder, generating a codeword of length \( n \) code bits with \( n - k \) parity check bits. This codeword is partitioned into \( M \) IR-frames according to an IR strategy defined as follows. Let \( t_i \) denote transmission \( i \), \( c_i \) denote the number of parity bits sent in transmission \( i \) and \( t_i : c_i \) denote that transmission \( t_i \) contains \( c_i \) parity check bits. Note that \( t_i \) will include \( k \) information bits and \( c_i \) parity check bits, whereas for \( i > 1 \), \( t_i \) includes only parity check bits according to:

\[
t_1 : k + c_1, t_2 : c_2, t_3 : c_3, \ldots, t_M : c_M
\]

such that

\[
\sum_{i=1}^{M} c_i \leq n - k, \quad \text{where} \quad c_i \in \{0,1,2,\ldots,n-k\}.
\] (2)

Following \( t_i \), the decoder attempts to decode the corresponding codeword of length

\[
n_i = k + \sum_{j=1}^{i} c_j.
\] (3)

The average code rate in an IR-HARQ scheme is a function of the retransmission criterion, and thus also a function of the probability of a retransmission, \( P_{ARQ}(n_i) \). Here, we assume that the criterion for declaring a retransmission is such that a received frame accepted based on a codeword of length \( n_i \) will also be accepted based on a codeword of length \( n_{i+1} \). Considering \( M = 2 \), we can transmit a codeword of length \( n_2 \) using at most two transmissions. The average number of code bits transmitted is then \( k + c_1 + c_2 P_{ARQ}(n_1) \). Generalizing this to \( M \) transmissions the code rate is given by

\[
r_{C_{ARQ=M}} = \frac{k}{k + c_1 + \sum_{i=2}^{M} c_i P_{ARQ}(n_{i-1})},
\] (4)

where \( c_i \) is subject to the constraints in (2) and \( n_i \) is given by (3).

The task is now to select \( c_i \), for \( i = 1,2,\ldots,M \) such that \( r_{C_{ARQ=M}} \) is maximized. For \( M \) transmissions, this is an \( M \) dimensional discrete maximization problem according to:

\[
[c_1', c_2', \ldots, c_M'] = \arg \max_{c_1,c_2,\ldots,c_M} r_{C_{ARQ=M}},
\] (5)

where \( r_{C_{ARQ=M}} \) is as given in (4) and \( c_i \) is subject to the constraints given in (2).

Having obtained the optimal number of parity bits to be included in each of the \( M \) incremental redundancy transmissions in order to maximize the average code rate for a given \( M \), we can obtain the maximum average code rate as a function of the number of transmissions, i.e., for \( M = 1,2,\ldots,n-k+1 \). Over all values of \( M \), the maximum average code rate is obtained by transmitting all \( k \) information bits in the first transmission and subsequently transmitting a single parity bit at a time, leading to \( M = n - k + 1 \) transmissions in total. With this strategy, each data frame is accepted following the transmission of a minimum number of parity bits, and the resulting average code rate is an achievable upper bound for any valid \( M = 1,2,\ldots,n - k + 1 \).

Numerical examples show that the average code rate benefits from typical diversity gain as a function of \( M \). For tested code families, significant increases are found for \( M = 2,3,4 \), while diminishing returns are observed for \( M > 4 \).

Additional constraints on error rate performance can also be imposed if a specific maximum target error rate is required over a range of signal to noise ratios. Only the number of parity bits necessary to achieve the specified target error rate are then included in the last transmission, allowing for an increase in the average code rate.

REFERENCES