

## Schwartz Inequality.

Schwartz inequality states that, given two vectors  $f$  and  $g$ , in a vector space  $V$  and a scalar product we have the inequality:

$$|\langle f, g \rangle| \leq \|f\| \cdot \|g\|$$

In this exercise, we apply Schwartz inequality to **Pattern recognition**.

By calculating

$$Q = \langle f, g \rangle / (\|f\| \cdot \|g\|)$$

as a **similarity measure** you can locate a given pattern (a small image) in a large image.

The pattern is represented as the matrix  $g$ , (here of odd size) whereas the matrix  $f$  represents a subpart of the image, equal in size with the pattern  $g$ .

### 1. Study the code for the MATLAB function `exsim`.

The MATLAB function `exsim` is included in this exercise. `Exsim` takes as **input parameters** an image  $I$ , and a pattern  $g$  of odd size. The **output** of the function is another image, the "**similarity map**"  $S$ . The value of each pixel in  $S$  is a measure of the similarity of the corresponding area of  $I$  to the pattern  $g$ . Similarity values are in the interval  $[0, 1]$ . You calculate  $S$  by the MATLAB command `S=exsim(A,g)`, where  $A$  is the input image,  $g$  is the matching pattern, and  $S$  is the output "similarity map".

Questions:

*Explain how the scalar product  $\langle f, g \rangle$  is calculated (fg in the code).*

*Explain how the norms  $\|f\|$  and  $\|g\|$  are calculated (norm\_f and norm\_g in the code). Notice that  $\|f\|$  is an image too, and that  $\|f\|^2$  is a scalar product between  $\langle f^2, \text{ones} \rangle$  where `ones` contains only 1s. Explain why the similarity image is in the interval  $[0, 1]$ .*

### 2. Pattern matching, synthetic image.

Create a  $128 \times 128$  image. The image should have a  $21 \times 21$  light square (intensity value 200) on a dark background (intensity value 40). Also create a matching pattern  $g$  as outlined below. Use the function `exsim` to calculate the similarity map.

At the MATLAB prompt, write:

```
A=ones(128,128).*40; %dark background, size 128 x 128
A(50:70,50:70)=ones(21,21).*200; %light square, size 21 x 21
%show the image in figure 1, and in gray
figure(1); subplot(2,2,1); imshow(A,[]); colormap(gray); axis image;
```

```
g=ones(21,21).*40; %pattern g
g(1:10,1:21)=ones(10,21).*200;
```

```
S=exsim(A,g); %calculate the similarity map
```

Questions:

*How does the pattern  $g$  look like?*

*Explain the position of the white dot (=most similar position) and that of the dark dot (=least similar position) in the original image  $A$ .*

*Produce a new pattern which is a rotation of  $g$  by 90 degrees.*

*Run `exsim` once again. Explain the result.*

### 3. Pattern matching, real image.

Load the image pout.tif. As the matching pattern g take a neighbourhood of the left eye.

```
B=imread('pout.tif'); %load the image
B=double(B); %datatype float
%show the image in figure1, and in gray
figure(1); colormap(gray);
subplot(2,3,1); imshow(B,[]); axis image;

g=B(80:100,108:128); %left eye as the matching pattern 21 x 21

D=B;
D(80:100,108:128)=ones(21,21).*150;
subplot(2,3,3); imshow(D,[]); axis image;%show where the pattern is located in the image

S=exsim(B,g);%calculate the similarity map
```

Questions:

*Can you find the white dot (=most similar position)?*

*Find out the coordinates of the white dot by the MATLAB command: [r,c]=find(S==max(max(S))).*

*Are there some parts of the image where similarity measures not calculated reliably or uniquely ? (Hint: attempt to calculate similarity close to a border)*

### 4. Tolerance to noise.

Add gaussian noise (zero-mean, variance=0.001) to the image pout.tif.

Do the pattern matching by running the MATLAB function exsim.

```
N=(imnoise(B/255,'gaussian',0,0.001)).*255; %add gaussian noise, mean=0, var=0.001
S=exsim(N,g); %calculate the similarity map
```

Questions:

*Find out if you still get a correct matching.*

*Increase the noise in step of 0.001 until the matching fails.*

*Which variance of the gaussian noise is tolerated for a correct matching?*