Registration of fingerprints by complex filtering and by 1D projections of orientation images

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Abstract

When selecting a registration method for fingerprints, the choice is often between a minutiae based or an orientation field based registration method. In selecting a combination of both methods, instead of selecting one of the methods, we obtain a *one modality multi-expert* registration system. If the combined methods are based on different features in the fingerprint, e.g. the minutiae points respective the orientation field, they are uncorrelated and a higher registration performance can be expected compared to when only one of the methods are used. In this paper two registration methods are discussed that do not use minutiae points, and are therefore candidates to be combined with a minutiae based registration method to build a multi-expert registration system for fingerprints with expected high registration performance. Both methods use complex orientations fields but produce uncorrelated results by construction. One method uses the position and geometric orientation of symmetry points, i.e. the *singular points* (SPs) in the fingerprint to estimate the translation respectively the rotation parameter in the Euclidean transformation. The second method uses 1D projections of *orientation images* to find the transformation parameters. Experimental results are reported.

1 Introduction

There are numerous techniques that use minutiae points in Automatic Fingerprint Identification Systems (AFIS) as well as low cost silicon sensor systems that are geared toward minutiae based techniques. This is due to long history of minutiae used in crime scene investigations. Consumer uses of biometrics increasingly questions the limitation of identification features to minutiae. Even more interestingly, by selecting a combination of features, instead of selecting minutiae, we can obtain a *one modality multi-expert* registration system. The two registration methods can be expected to be uncorrelated if they are based on different features in the fingerprint, e.g. the minutiae pattern respective the orientation field. By combining the output of uncorrelated methods a gain in the registration performance can be achieved, compared to the use of only one of the methods. This is because the methods complement each other in a positive way. When one method fails the other may still have success in the registration.

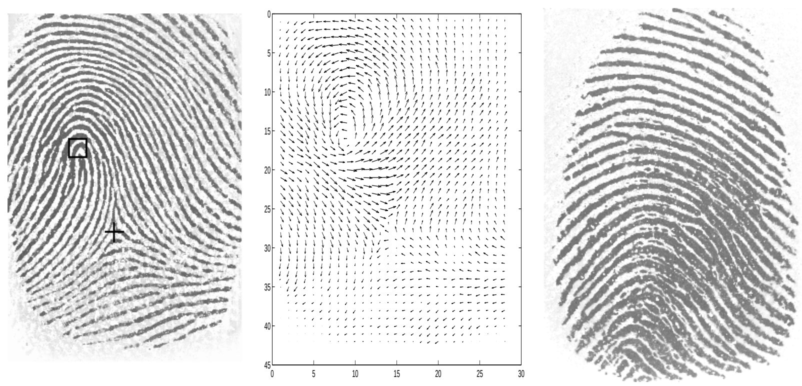

Fig. 1. Left: marked singular points, a core point is marked with a square and a delta point with a cross. Middle: the estimated complex orientation field at level 3 for the fingerprint to the left. Right: a fingerprint of type arch.

In the minutiae based registration methods the fingerprints are represented by its minutiae points, i.e. the position and the orientation of their minutiae are elements in their respective feature vector representation. Aligning the two fingerprints is to find the transformation parameters that maximize the number of matching minutiae pairs in the feature vectors [1, 2]. If the transformation is the Euclidean transformation, the parameters are the rotation angle and the translation vector [3] relating the template and the test fingerprint.

However in low quality fingerprints it is difficult to automatically extract the minutia points in a robust way. This often means that genuine minutiae are missed and that false minutiae are added [2]. Also, in cost sensitive applications, because the price of the sensor depends on the sensor area, sensors with small areas are used and therefore fewer numbers of minutiae are present in the captured fingerprint. For these two situations a high performance registration is difficult to obtain if only the minutiae based registration method is used. A higher performance can be expected if the minutiae based method can be combined with an other technique which we suggest to be orientation field features.

In this paper two registration methods are suggested that use the global structure of the fingerprint, and therefore are more robust to low quality fingerprint registration and more suitable to register fingerprints captured from small area sensors. They are therefore candidates to be combined with a minutiae based registration method to build a multi-expert registration system for fingerprints as discussed above. One method uses the position and geometric orientation of symmetry points, i.e. the singular points (SPs) in the fingerprint (see Figure 1) to estimate the translation respectively the rotation parameter in the Euclidean transformation [4]. The second method uses 1D projections of orientation images [5] to find the transformation parameters intended for a situation when SPs are poorly imaged. Both methods complement each other as well as minutiae and used complex orientation fields (see Figure 1).

2 Registration by symmetry points

This method (called method 1) extracts automatically the position and the geometric orientation of SPs, from the global structure using complex filters designed to detect rotational symmetries. The translation is estimated from the difference in position, and the rotation parameter from the difference in the geometric orientation of SPs in the test and the template fingerprint. In [4] we have shown that an unbiased alignment error with a standard deviation of approximately the size of the average wavelength (13 pixels) of a fingerprint is possible to achieve using this method.

A common technique to extract SPs (core and delta points) in fingerprints is to use the *Poincaré* index introduced by Kawagoe and Tojo [6]. It takes the values $180°$, $-180°$, and $0°$ for a core point, a delta point, and an ordinary point respectively. It is obtained by summing the change in orientation following a closed curve counterclockwise around a point [7]. This technique has been used in the studies of Karu and Jain [7], and Bazen and Gerez [8] to define and extract SPs.

Our method using complex filters compared to *Poincaré* index to identify SPs has the advantage to extract not only the position of an SP but also its spatial orientation. When two fingerprints are rotated and translated relative to each other our method can estimate both translation and rotation parameters simultaneously. In the work of Bazen and Gerez [8] the position extraction and the orientation estimation of an SP is done in two sequential steps. The position extraction is performed by using *Poincaré* index. The orientation estimation is done by matching a reference model of the orientation field around an SP with the orientation map of the extracted SP. The orientation maps were obtained by using a technique introduced in [9].

2.1 Filters for rotational symmetry detection

Complex filters, of order m, for the detection of patterns with rotational symmetries are modeled by $e^{im\varphi}$ [10, 11]. A polynomial approximation of these filters in gaussian windows yields $(x+iy)^m g(x,y)$ where g is a gaussian defined as $g(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$ [12, 13].

It is worth to note that these filters are not applied to the original fingerprint image but instead they are applied to the complex valued orientation field image $z(x,y) = (f_x + if_y)^2$. Here f_x is the derivative of the original image in the x-direction and f_y is the derivative in the y-direction.

In our experiments we use filters of first order symmetry or parabolic symmetry i.e.
$h_1(x,y) = (x+iy)g(x,y) = re^{i\varphi}g(x,y)$ and
$h_2(x,y) = (x-iy)g(x,y) = r\ e^{-i\varphi}g(x,y) = h_1^*$.
Patterns that have a local orientation description of $z = e^{i\varphi}$ (m=1) and $z = e^{-i\varphi}$ (m=-1) are shown in Figure 2. As can be seen these patterns are similar to patterns of a core respectively a delta point in a fingerprint and therefore suitable to

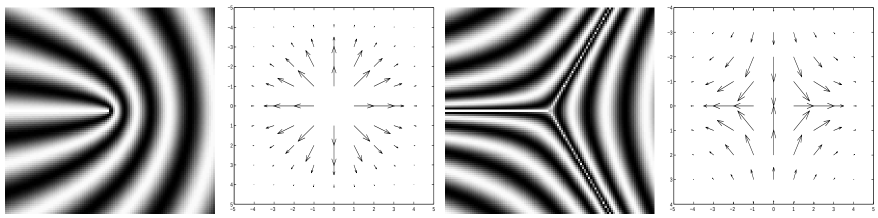

Fig. 2. Patterns with a local orientation description of $z = e^{i\varphi}$ (left) and $z = e^{-i\varphi}$ (right). Both in gray scale (patterns) and in z representation (complex filters).

use as SP-extractors. The SP-extractors are the z representation of the patterns, i.e. the complex filter h_1 and h_2 respectively.

The complex filter response is $c = \mu \ e^{i\alpha}$, where $\mu = \frac{|I_{20}|}{I_{11}}$ is a certainty measure of symmetry, and $\alpha = Arg(I_{20})$ is the "member" of that symmetry family, here representing the geometric orientation of the symmetric pattern. The scalars $I_{20} = \langle h_1, z \rangle$ for the core point extraction, $I_{20} = \langle h_2, z \rangle$ for the delta point extraction, and $I_{11} = \langle |h_1|, |z| \rangle$ are obtained by use of the 2D complex scalar product symbolized by $\langle \ \rangle$ [12]. Representing the certainty measures by μ_1 and μ_2 for core point respectively delta point symmetry, we can identify an SP of type core if $\mu_1 > T_1$ and of type delta if $\mu_2 > T_2$, where T_1 and T_2 are empirically determined thresholds.

2.2 Multi-scale filtering

Using a multi-resolution representation of the complex orientation field offers a possibility to extract SPs more robustly and precisely compared to a representation at only one resolution level. The extraction of an SP starts at the lowest resolution level (a smooth orientation field) and continues with refinement at higher resolutions. The result at a low resolution guides the extraction at higher resolution levels. This strategy can be taken because SPs have a global support from the orientation field [14].

The complex orientation field $z(x, y)$ is represented by a five level Gaussian pyramid. Level 4 has the lowest, and level 0 has the highest resolution. The core and the delta filtering is applied on each resolution. The complex filter response is called c_{nk}, where k=4, 3, 2, 1 and 0 are the resolution levels, and n=1, 2 are the filter types (core and delta).

Figure 3 (upper row) shows the magnitude of the filter responses of filter h_1 (called μ_{1k}). The filter is applied to the orientation field of the fingerprint to the left in Figure 1.

2.3 Multi-scale searching for SPs

The extraction of an SP starts at the lower resolution level, i.e. we search for maximum in the certainty image μ_1 and μ_2 for a core and a delta point respectively. In the found position of maximum $(x, y)^{max}_{n4}$ we extract the complex filter

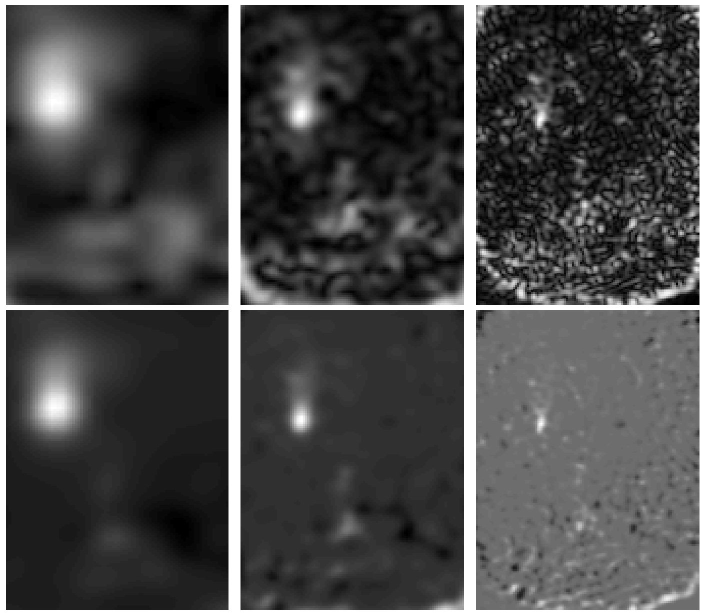

Fig. 3. Filter responses for the fingerprint to the left in Figure 1, core point. Row1: μ_{1k}, k=3, 2, and 1. Row2: enhanced μ_{1k}^{enh}, k=3, 2, and 1.

response c_{n4}^{max}, for each type of SP, which is a vector pointing in the geometric orientation of respective SP. The magnitude of this vector is put to one, we call this vector SP_{or} of which there is one per resolution level and SP type. The SP_{or} is then used to define the *search window* (for a core point only) and to increase the signal-to-noise ratio in the certainty images μ_1 and μ_2 when searching for maximum at the next higher resolution level. More precisely, the enhanced certainty image μ_{k-1}^{enh} at level $(k-1)$ is obtained according to equation 1, where φ is the difference in angle between a filter response vector c_{k-1} and the SP_{or} vector at previous lower resolution level k.

$$\mu_{k-1}^{enh} = \mu_{k-1} \cdot cos(\varphi) \tag{1}$$

The quantity μ_{k-1} represents the certainty as described in section 2.1 and the above equation is a vectorial projection of c_{k-1} on SP_{or}. In this way we lower the responses of those complex filter responses that are not coherent with the orientation of the SP_{or} at the previous lower resolution level. Figure 3 (lower row) shows the enhanced certainty image for a core point for the fingerprint to the left in Figure 1. This is repeated for each search of maximum between levels in the Gaussian pyramid.

At each level k we extract in the complex filter response image c_{nk} at the position $(x, y)_{nk}^{max}$ found in the enhanced certainty image μ_k^{enh}. We call these complex filter responses c_{nk}^{max}.

3 Registration by 1D projections of orientation images

One class of fingerprints, i.e. class arch (see Figure 1 to the right), lacks SPs [7]. In noisy fingerprints the complex filtering can give a too weak response to classify the point as a core or a delta point. Also when the sensor area of the capturing device is small the SPs are not always found within the captured fingerprint. In these situations symmetry point extraction will fail and must be complemented by an alternative method. We call this method "Registration by 1D projections of orientation images" which makes use of the global orientation field of the fingerprint but does not need SPs for registration. The method is based on a decomposition of the fingerprint into several images, where each image, O_k, corresponds to a direction. Called *Orientation images* in what follows, they were 6 in number, representing 6 equally spaced directions in our experiments.

By *a pair of Orientation images* we mean two orientation images, one from the template fingerprint and one from the test fingerprint, belonging to the same orientation value. The difference in position of a pair of orientation images, is used to estimate the translation between the template and the test fingerprint (it is assumed that the rotation is negligible, or have been compensated for, between the two fingerprints). From each of the orientation images several 1D projections at different angles (*radiograms*) are computed [5]. We call the two radiograms computed from a pair of orientation images at the same projection angle *a pair of radiograms*. A correlation is computed between each pair of radiograms. From the peak in the correlation measure we estimate a displacement for each such pair of radiograms.

In the estimation of the translation parameter we make use of two displacements computed from pair of radiograms which are perpendicular in projection angle. The final estimate of the translation between the template and the test fingerprint is computed from the total of $n_{or} * \frac{n_{pr}}{2}$ number of estimates, where n_{or} and n_{pr} are the number of orientation images and the number of projection angles respectively.

3.1 Orientation radiograms

An orientation image is computed according to equation 2.

$$O_k = |z|\, e^{\alpha(cos^2(\theta_k - \varphi)-1)} \tag{2}$$

In this equation Θ_k is the pass orientation angle for an orientation image O_k and φ is the orientation of the orientation field z in each point in the interval $\left[-\frac{\pi}{2}\ \frac{\pi}{2}\right]$. The constant α controls the sensitivity in the selection of orientation angles close to the pass angle. Figure 4 shows orientation images for pass angles $-\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ at level 2 in the Gaussian pyramid when the input is the fingerprint given in Figure 1 to the right.

The Radon transform is used to compute 1D projections of orientation images in the direction of ϕ according to equations 3 and 4. Figure 4 shows radiograms

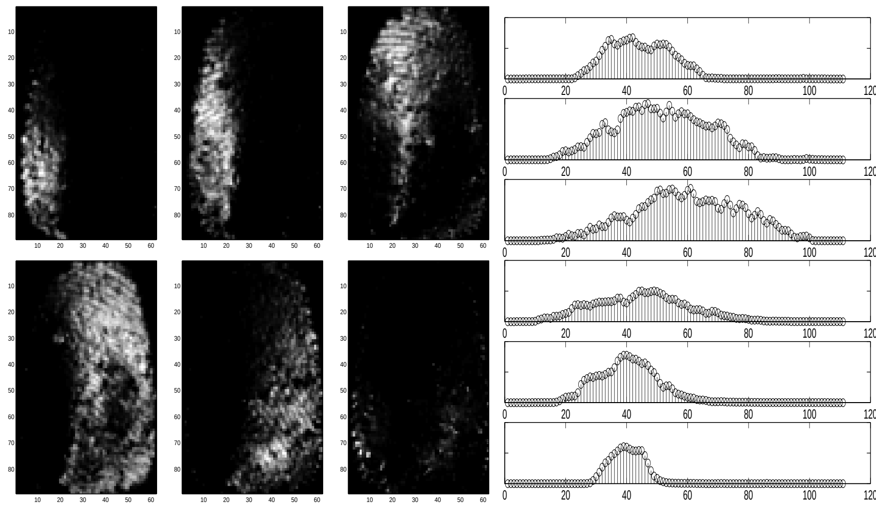

Fig. 4. Left: Orientation images (level 2) for the fingerprint to the right in Figure 1. With pass angles $\theta_k = -\frac{\pi}{3}, -\frac{\pi}{6}, 0$ for the upper row from left to right, and $\theta_k = \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ for the bottom row from left to right. Right: Radiograms computed from the orientation image to the left in this figure with pass angle $-\frac{\pi}{6}$. Projection angles from top to bottom are $-\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$.

for the orientation image to the left with a pass angle of $-\frac{\pi}{6}$. Radon transform amounts to summing the pixel values along the direction ϕ.

$$R_\phi\left(x'\right) = \int f\left(x' cos\phi - y' sin\phi, x' sin\phi + y' cos\phi\right) dy' \tag{3}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\phi & sin\phi \\ -sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{4}$$

3.2 Translation estimation

The relation between the displacement dx' between a pair of orientation radiograms and the translation dx and dy of its pair of orientation images is computed according to equation 5.

$$dx'_{\phi_m} = cos\phi_m dx + sin\phi_m dy \tag{5}$$

Where $dx' = x'_{template} - x'_{test}$, $dx = x_{template} - x_{test}$, and $dy = y_{template} - y_{test}$.

The displacement dx' is estimated from data of a certain projection angle ϕ_m by finding the peak in the correlation signal of each pair of orientation radiograms. By using two pairs of radiograms, which are perpendicular in projection angle, we can estimate the translation dx and dy between the template fingerprint and the test fingerprint by equation 6.

$$\begin{bmatrix} dx'_{\phi_m} \\ dx'_{\phi_n} \end{bmatrix} = \begin{bmatrix} cos\phi_m & sin\phi_m \\ -sin\phi_m & cos\phi_m \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} \tag{6}$$

Where $\phi_m = -\frac{\pi}{3}, -\frac{\pi}{6}, 0$ and $\phi_n = \phi_m + \frac{\pi}{2}$, i.e. $\frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$. From each pair of orientation images we get $\frac{n_{pr}}{2}$ number of estimates. Out of a total of $n_{or} * \frac{n_{pr}}{2}$ estimates we want to select, in a robust way, the final translation estimate dx, dy. First we apply an outlier detection within an orientation image by disregarding estimates $(dx, dy)^T$ that are most dissimilar to other estimates. Second we take away orientation images that have a high variance in their estimates. Finally we estimate the translation by taking the mean value of the estimates.

4 Experiments

The FVC2000 fingerprint database, db2 set A [15] is used in the experiments. A total of 800 fingerprints (100 persons, 8 fingerprint/person) are captured using a low cost capacitive sensor. The size of an image is 364 x 256 pixels, and the resolution is 500 dpi. It is worth to note that FVC2000 is constructed for the purpose of grading the performance of fingerprint recognition systems, and contains many poor quality fingerprints.

4.1 Symmetry point extraction

The filters used in the multi-scale filtering are of size 11 x 11 (a standard deviation of the Gaussian of 1.6). From the multi-scale searching for maximum in the enhanced certainty images μ_{nk}^{enh}, as described in section 2.3, the position of maximum $(x, y)_{nk}^{max}$ is extracted for each level k and for each type n of SP (in the lowest resolution level the search for maximum is done in the ordinary certainty image μ_{nk}). In the position $(x, y)_{nk}^{max}$ the complex filter responses c_{nk}^{max} are extracted and saved for each level k and for each type n of SP. We compute new filter responses R from the extracted complex filter responses c_{nk}^{max} according to equations 7 and 8, i.e. we sum the complex filter responses c_{nk}^{max} in the levels k (vector-sum) for respective type of SP. The final response is the mean of the magnitude of the vector-sum.

$$R_{core} = \frac{1}{4} \left| \sum_{k=1}^{4} c_{1k}^{max} \right| \quad (7)$$

$$R_{delta} = \frac{1}{3} \left| \sum_{k=1}^{3} c_{2k}^{max} \right| \quad (8)$$

To test the performance of the symmetry point extraction (method 1) the true position $(x, y)_n^{true}$ of the SPs have been manually extracted for the fingerprints in the database. Those fingerprints that are lacking SPs are marked manually for being so. An SP is "correct in position" if the Euclidean distance d between the true position and the extracted position at resolution level 1 $(x, y)_{n1}^{max}$, by method 1, is ≤ 15 pixels [1], and the filter responses R_{core} respectively R_{delta} are

[1] approximately 1.5 wavelength of the fingerprint pattern

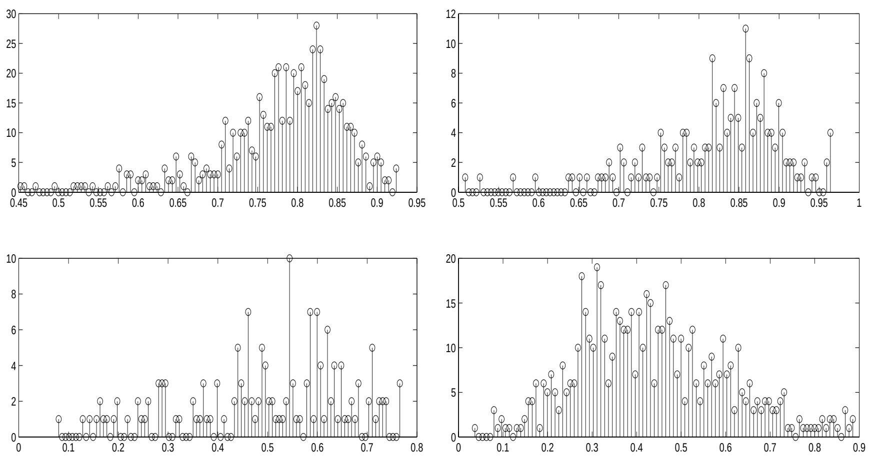

Fig. 5. Distributions of R_{core} (left) and R_{delta} (right). Left: core points that are "correct in position" (top), and core points that are "not correct in position" (bottom). Right: delta points that are "correct in position" (top), and delta points that are "not correct in position" (bottom).

high, i.e. higher than a threshold. Figure 5 shows the distribution of the filter response R_{core} for "correct in position" extracted core points (left/top) and for core points "not correct in position" (left/bottom) and the distribution of the filter response R_{delta} for "correct in position" extracted delta points (right/top) and for delta points "not correct in position" (right/bottom). From these distributions we can estimate the performance for method 1 for different values of thresholds.

If we put the threshold for core point acceptance $th_{core} = 0.63$ we get an EER of 4 % and for the delta points we get an EER of 3 % when the threshold for acceptance for a delta point $th_{delta} = 0.73$. In 665 fingerprints out of 800 we find a core point, or a delta point, or both "correct". By correct we mean both close in distance to the true position (closer than 15 pixels) and a high response from the symmetry filter, i.e. $R_{core} > th_{core}$ respective $R_{delta} > th_{delta}$.

In figure 6 the histograms of the error in distance for the "correctly" estimated SPs are shown. The mean value of the error in distance is approximately $5 \ \pm 3$ pixels.

4.2 Orientation radiograms

We apply method 1 to obtain SPs. For those fingerprints which does not contain sufficiently strong SPs we apply the alternative method discussed in section 3. Method 1 finds a symmetry point, in 665 fingerprints out of 800. The method 2 is tested on the remaining 135 fingerprints. We call these 135 fingerprints the *SP-free set*. We use a jack-knife strategy to measure the performance of method 2, using the rotation principle because we rotate the test data with the template

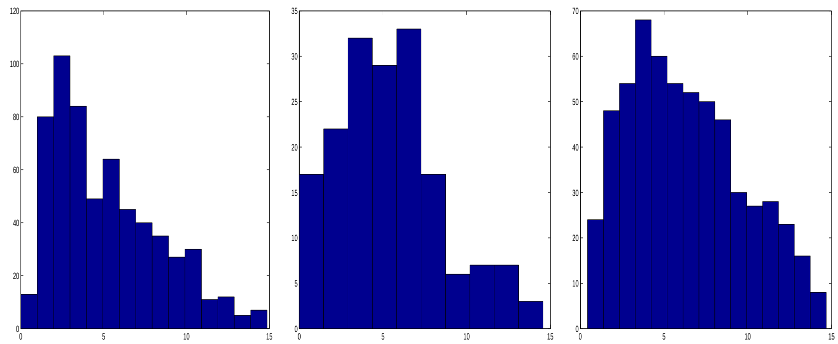

Fig. 6. Histograms of the error in distance for "correct" SPs. To the left for core points and in the middle for delta points. The mean error in distance is approximately 5 ± 3 pixels for both type of SPs. To the right the histogram of the error for the "correctly" estimated translation parameters. The mean error is approximately 6 ± 3 pixels.

data to obtain the maximum available trials. This is motivated by that method 1 leaves too few samples for method 2 to work on despite the fact that the size of the FVC database is appreciably large. For each fingerprint in the SP-free set (the test fingerprint) we estimate the translation parameters by using the rest of the fingerprints for that person as template fingerprints. The templates may or may not have been found by method 1. In this way we obtain 7 estimates for each test fingerprint, that is a total of $7 * 135 = 945$ translation estimates.

In the experiments we have used 6 orientation images n_{or} with pass angles $-\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ and 6 projection angels n_{pr} equal in value to the pass angles. This gives 3 estimates for each orientation pair, and a total of 18 translation estimates. The orientation images O_k are computed using the orientation field z at level 1 in the Gaussian pyramid, the parameter α found empirically is put to 8.6 in equation 2.

In the processing of the 18 estimates of $(dx, dy)^T$ we obtain new estimates stemming from within and between orientation images. First, within an orientation image, we take away one estimate out of 3. The one which is most dissimilar to the other two is disregarded. Second we keep 3, i.e. the 3 orientation images which have minimum variance in their estimates. Third we keep the two orientation images that are closest in the mean of their estimates. Now we have two orientation images, and two estimates of translation for each object. Fourth we take as the final estimate the mean of the two estimates belonging to the orientation image that shows minimum variance.

Figure 7 shows the result. The distance metric is the Euclidean distance between the true translation and the estimated translation. If we assume that the error in the translation estimate is acceptable if the euclidean distance $d \leq 15$ pixels (we name this "correct" estimation) the above method finds the true translation in 588 trials out of 935 possible trials. In figure 6 the histogram of

FVC2000 database, 935 trials in the test.	Number of fingerprints	Ratio %
$distance \leq 15$	588	62
$distance \leq 20$	647	68
$distance \leq 30$	716	76

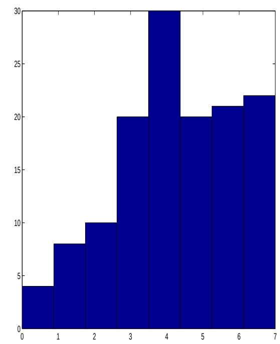

Fig. 7. Results from method 2. Left: Distances from true translation. Right: Histogram number of outcomes (maximum =7 and minimum=0).

the error for the "correctly" estimated translation parameters is shown. The mean value of the error is approximately 6 ± 3 pixels.

For each test fingerprint (a total of 135 tests, 7 estimates in each test) the possible outcomes that are "correct", i.e. $d \leq 15$, is in the range [0 7]. To the right in Figure 7 is the histogram for this test. The mean value is 4.3 which means that in the mean approximately 4 estimates out of 7 are "correct" for each test fingerprint by using method 2 in isolation on fingerprints that are rejected by method 1.

4.3 Registration using the combined methods

Using method 1 we detect the position of an SP (core or delta, or both) in 665 fingerprints out of 800 with an acceptable error in distance of 5 ± 3 pixels. In [4] we have shown that using SP-registration in isolation an unbiased alignment error with a standard deviation of 13 pixels (which approximately is the average wavelength in the fingerprint) can be achieved. We also present a performance measure of the estimation of the geometric orientation of an SP to be unbiased with a standard deviation of less than 4°. Using SP-registration with the 665 correctly extracted SPs, and assuming the same alignment error as in [4], we achieve a registration performance of 83% for SP-registration in isolation.

The alternative method (method 2) running on the 135 fingerprints missed by method 1 estimates correctly 588 trials out of 945 possible trials (62%) with a mean error of 6 ± 3 pixels. With this performance for method 2, we estimate the translation parameter in an acceptable way for 84 fingerprints of 135, missed by method 1. The 135 fingerprints were not compensated for orientation differences. However, for the 84 fingerprints in which a "correct" translation estimate was found, the orientation difference is small (because of how the translation estimation was implemented) and therefore also the rotation difference is small. Accordingly, it can be concluded that a registration performance of 62% is achieved with this method in isolation with an alignment error of similar order as for method 1.

To conclude, by using method 1 and method 2 jointly we estimate the translation parameter "correctly" for 749 $(665 + 84)$ fingerprints out of total 800, yielding an identification performance of 94%. This is done without use of minutiae, and without rotation compensation for method 2.

5 Conclusion and Future work

In this paper a multi-expert registration system is built using non-minutiae features which makes the suggested method fully complementary to minutiae based methods.

The registration performance for the method *registration by symmetry points* was 83% when running in isolation. Combined with the method *registration by 1D projections of orientation images* the registration performance was increased to 94%. This shows that a combination of registration methods, i.e to use a *one modality multi-expert* registration system, instead of using one registration method in isolation increase the system registration performance. The achieved uncertainty (one standard deviation) of 13 pixels in the alignment error is approximately of the same size as other studies used, e.g. [16].

The 94% performance in the estimation of the translation parameter was achieved when the fingerprints for method 2 were not compensated for rotation differences. Before estimating the translation we can compensate for the rotation differences between the test and the template orientation images by a rough orientation estimation technique, such as orientation histogram correlation. This should increase the performance of registration for method 2.

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